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# Parity Nonconservation in Electromagnetic Systems

**E. Comay**[1]

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It is shown that a correlated motion of a charge–monopole system emits radiation that violates parity conservation.

It is well known that ordinary electrodynamics of charges and fields conserves parity. In particular, fields of an electromagnetic source which is invariant under parity transformation $\mathbf{r} \to -\mathbf{r}$ satisfy parity invariance. The intensity of radiation $\mathbf{S} = \mathbf{E} \times \mathbf{B}/4\pi$ emitted from this kind of source satisfies

$$\mathbf{S}(\mathbf{R}) = -\mathbf{S}(-\mathbf{R}) \tag{1}$$

at every point $\mathbf{R}$. An example of such a system is a circular antenna whose center is at the origin. This source is invariant under parity transformation and a time-dependent current along the circle emits radiation that satisfies (1). The objective of the present work is to prove that one can perform thought experiments with a charge–monopole system that violates parity invariance. In the system discussed, the motion of the charge alone conserves parity. The same is true for the motion of the monopole. However, the interference part of the radiation emitted from the correlated motion of these particles is inconsistent with parity conservation.

Consider a system of a single charge $Q$ attached to the circumference of a disk which rotates in the $(x, y)$ plane around the $z$ axis (see Figure 1a). Let $r = 1$ and $\omega$ denote the disk's radius and its angular velocity, respectively. The disk is made of an insulating material and its electric and magnetic susceptibilities are the same as those of the vacuum. Calculations are carried out in units where $c = 1$. Greek indices range from 0 to 3. The metric is diagonal and its entries are $(1, -1, -1, -1)$. The nonrelativistic limit holds

[1]School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel.

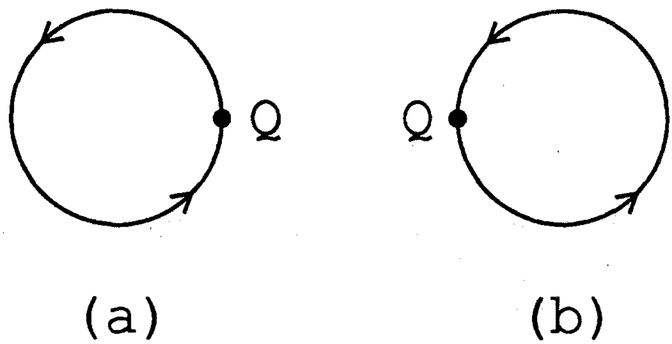


**Fig. 1.** (a) A disk rotates around the $z$ axis, carrying a charge $Q$. (b) The result of a parity transformation applied to the system depicted in (a).

and the charge's velocity satisfies

$$v = \omega \ll 1 \tag{2}$$

The Lienard–Wiechert fields of the rotating charge are

$$\mathbf{E} = Q\left[\frac{1-v^2}{(R-\mathbf{R}\cdot\mathbf{v})^3}(\mathbf{R}-R\mathbf{v}) + \frac{1}{(R-\mathbf{R}\cdot\mathbf{v})^3}\mathbf{R}\times\langle(\mathbf{R}-R\mathbf{v})\times\mathbf{a}\rangle\right] \tag{3}$$

$$\mathbf{B} = \mathbf{R}\times\mathbf{E}/R \tag{4}$$

where all quantities on the right-hand sides are taken at the retarded time. $\mathbf{R}$ denotes the radius vector from the charge's location to the point where the fields are measured. $\mathbf{v}$ and $\mathbf{a}$ denote the charge's velocity and acceleration, respectively (see, e.g., Landau and Lifshitz, 1975, p. 162).

Let us look at two points $\mathbf{P}_i = (0, 0, \pm z_0)$ on the $z$ axis at the wave zone. Thus, the relation

$$\omega z_0 \gg 1 \tag{5}$$

holds. The $z$ component of the Poynting vector of the radiation emitted by the rotating charge and measured at these points is calculated.

The first term of (3) behaves like $r^{-2}$, whereas the second one decreases like $r^{-1}$. Hence, using (5), one obtains the well-known result which says that the first term of (3) is ignored at the wave zone. Let $t=0$ be the retarded time of the charge with respect to the time when the fields are measured at $\mathbf{P}_i$. The charge's kinematic variables at $t=0$ are

$$\mathbf{r} = (1, 0, 0) \tag{6}$$

$$\mathbf{v} = (0, \omega, 0) \tag{7}$$

$$\mathbf{a} = (-\omega^2, 0, 0) \tag{8}$$

Hence, at the instant considered, the retarded radius vector is

$$\mathbf{R}_i = (-1, 0, \pm z_0) \simeq (0, 0, \pm z_0) \tag{9}$$

where the positive sign refers to $\mathbf{P}_1$ and the negative one refers to $\mathbf{P}_2$.

Substituting these values into the second term of (3) and (4) and using (2) and (5), one finds

$$E_x(\mathbf{P}_i) \simeq Q\omega^2/z_0 \tag{10}$$

$$E_y(\mathbf{P}_i) \simeq 0 \tag{11}$$

$$B_x(\mathbf{P}_i) \simeq 0 \tag{12}$$

$$B_y(\mathbf{P}_i) \simeq \pm Q\omega^2/z_0 \tag{13}$$

Here, as in (9), the positive sign refers to $\mathbf{P}_1$ and the negative one refers to $\mathbf{P}_2$. These results yield the required $z$ components of the Poynting vector at $\mathbf{P}_i$,

$$S_z(\mathbf{P}_i) \simeq \pm Q^2\omega^4/z_0^2 \tag{14}$$

where the positive and negative signs apply as in (9) and (13).

This expression proves that the nonnegligible part of the Poynting vector at $\mathbf{P}_i$ is directed outward and represents radiation emitted by the source. Using the cylindrical symmetry of the system, one finds that (14) is time-independent.

It is evident that these results provide an example of parity conservation. This property is shown in detail in a covariant way which will be helpful in a later discussion of the monopole case. Subscripts $(e)$ and $(m)$ denote quantities related to charges and monopoles, respectively. The field tensor

$$F^{\mu\nu}_{(e)} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \tag{15}$$

is used in a formulation of the charge's 4-acceleration, as expressed by the Lorentz force

$$m_{(e)} a^{\mu}_{(e)} = Q F^{\mu\nu}_{(e)} v_{(e)\nu} \tag{16}$$

where $m_{(e)}$, $v_{(e)}$, and $a_{(e)}$ denote the charge's mass, 4-velocity, and 4-acceleration, respectively. The 4-velocity is

$$v^{\mu}_{(e)} = \gamma(1, v_x, v_y, v_z) \tag{17}$$

and $\gamma = (1 - v^2)^{-1/2}$.

The parity operator takes the form

$$P^{\mu}_{\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} \tag{18}$$

Let us see how a 4-vector transforms under $P^{\mu}_{\nu}$. Applying it to the 4-velocity (17), one obtains

$$P^{\mu}_{\nu}v^{\nu}_{(e)}=\gamma(1, -v_x, -v_y, -v_z) \tag{19}$$

This result shows that the spatial components of a 4-vector change sign under parity transformation whereas its time component remains as is. Let us turn to the parity transformation of tensors. As an example, the fields tensor (15) is examined. Here one finds

$$P^{\mu}_{\alpha}P^{\nu}_{\beta}F^{\alpha\beta}_{(e)}=\begin{pmatrix}0 & E_x & E_y & E_z\\ -E_x & 0 & -B_z & B_y\\ -E_y & B_z & 0 & -B_x\\ -E_z & -B_y & B_x & 0\end{pmatrix} \tag{20}$$

This result means that the components (0, 1), (0, 2), and (0, 3) and their symmetric ones change sign, whereas all other components of a tensor are left unchanged. If in the case of the antisymmetric tensor (15) one wishes to use a 3-dimensional terminology, one may say that the electric field transforms like a vector and the magnetic field transforms like an axial vector.

The components of the Poynting vector $\mathbf{S}$ are the (0, 1), (0, 2), and (0, 3) entries of the field's energy-momentum tensor (see, e.g., Landau and Lifshitz, 1975, p. 81)

$$T^{\mu\nu}_{(f)}=\frac{1}{4\pi}\left(F^{\mu\alpha}F^{\beta\nu}g_{\alpha\beta}+\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}g^{\mu\nu}\right) \tag{21}$$

Evidently, the Poynting vector transforms like a vector.

These general results can be applied to the specific experiment discussed here. The position of the charge at $t=0$ transforms as follows: $\mathbf{r}(0)=(1, 0, 0)\rightarrow(-1, 0, 0)$ (see Figure 1b). The kinematic variables $\mathbf{v}$ and $\mathbf{a}$ undergo a similar transformation. It is clear that a parity transformation of the rotating charge takes it to a state which is precisely the same as its state at $t=\pi/\omega$. Therefore, time-independent quantities associated with the source can be used in a test of parity conservation. A parity transformation of the points $\mathbf{P}_i$ is $\mathbf{P}_1\leftrightarrow\mathbf{P}_2$. As shown above, the same relation holds for the two Poynting vectors $\mathbf{S}_i$ at $\mathbf{P}_i$. These results are consistent with parity conservation by the system.

The second system is like the first one, but a monopole replaces the rotating charge. As before, we are interested in the fields at the two points $\mathbf{P}_i$ at the wave zone.

The electrodynamics of a system of monopoles is obtained from that of charges by means of the duality transformations (Goddard and Olive, 1978)

$$F^{*\mu\nu}=\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}g_{\alpha\gamma}g_{\beta\delta}F^{\gamma\delta}=\begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix} \tag{22}$$

$$e\to g; \qquad g\to -e \tag{23}$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric unit tensor of the fourth rank.

These transformations yield the Lienard–Wiechert fields of a monopole

$$\mathbf{B}=g\left[\frac{1-v^2}{(R-\mathbf{R}\cdot\mathbf{v})^3}(\mathbf{R}-R\mathbf{v})+\frac{1}{(R-\mathbf{R}\cdot\mathbf{v})^3}\mathbf{R}\times\langle(\mathbf{R}-R\mathbf{v})\times\mathbf{a}\rangle\right] \tag{24}$$

$$\mathbf{E}=-\mathbf{R}\times\mathbf{B}/R \tag{25}$$

The fields (22) are related to the monopole dynamics in a covariant way, as (15) was used in (16),

$$m_{(m)}a^{\mu}_{(m)}=QF^{*\mu\nu}_{(m)}v_{(m)\nu} \tag{26}$$

The parity properties of each element of a monopole system are obtained from this covariant form of monopole dynamics in an analogous way to that used in the derivation of (19) and (20). In particular, an examination of the tensor (22) and the force (26) shows that, in the monopole world, magnetic fields transform like vectors and electric fields transform like axial vectors. These properties are used in the evaluation of the second experiment.

The fields of the rotating monopole at $\mathbf{P}_i$ are obtained from (24) and (25) by arguments which are analogous to the ones used previously for the charge. The same is true for the Poynting vector. Under the parity transformation (18), one obtains analogous quantities. It follows that the monopole, like the charge of the previous system, emits radiation that satisfies parity conservation.

The two experiments described above are elements of the main experiment discussed in this work. Consider a system made up of a charge $Q$ and a monopole $g$, where

$$g=Q \tag{27}$$

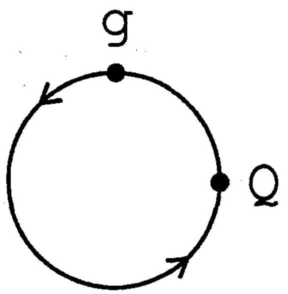


**Fig. 2.** A disk rotates around the $z$ axis. A charge $Q$ and a monopole $g$ are attached to the disk's circumference at points that make an angle of $\pi/2$.

These particles are attached to a rotating disk like the one discussed above (see Figure 2). Let $t=0$ be the retarded time. At this instant, the charge is at $\mathbf{r}_1=(1, 0, 0)$ and the monopole is at $\mathbf{r}_2=(0, 1, 0)$. As before, we are interested in the radiation fields at the two points $\mathbf{P}_i$ at the wave zone. Relying on the linearity of classical electrodynamics, one can readily use the fields associated with the charge as obtained earlier in (10)–(13). Due to the cylindrical symmetry of the problem, one finds the same retarded time for the charge and the monopole with respect to $\mathbf{P}_i$. At $t=0$, the monopole's kinematic variables are

$$\mathbf{r}_{(m)}=(0, 1, 0) \tag{28}$$

$$\mathbf{v}_{(m)}=(-\omega, 0, 0) \tag{29}$$

$$\mathbf{a}_{(m)}=(0, -\omega^2, 0) \tag{30}$$

Using (24)–(30) and making the same approximations as before, one finds that, at $\mathbf{P}_i$, the fields associated with the monopole $g$ are

$$B_x(\mathbf{P}_i)\simeq 0 \tag{31}$$

$$B_y(\mathbf{P}_i)\simeq g\omega^2/z_0 \tag{32}$$

$$E_x(\mathbf{P}_i)\simeq \pm g\omega^2/z_0 \tag{33}$$

$$E_y(\mathbf{P}_i)\simeq 0 \tag{34}$$

In (33), as in earlier cases, the positive sign refers to $\mathbf{P}_1$ and the negative one refers to $\mathbf{P}_2$.

The fields (31)–(34), together with (10)–(13), yield the overall fields at $\mathbf{P}_i$. Using (27), one finds at $\mathbf{P}_1$

$$E_x(\mathbf{P}_1) \simeq 2Q\omega^2/z_0 \tag{35}$$

$$E_y(\mathbf{P}_1) \simeq 0 \tag{36}$$

$$B_x(\mathbf{P}_1) \simeq 0 \tag{37}$$

$$B_y(\mathbf{P}_1) \simeq 2Q\omega^2/z_0 \tag{38}$$

On the other hand, the combined fields at $\mathbf{P}_2$ are

$$\mathbf{E}(\mathbf{P}_2) \simeq \mathbf{B}(\mathbf{P}_2) \simeq 0 \tag{39}$$

Thus, the corresponding $z$ components of the Poynting vectors are

$$S_z(\mathbf{P}_1) \simeq 4Q^2\omega^4/z_0^2 \tag{40}$$

$$S_z(\mathbf{P}_2) \simeq 0 \tag{41}$$

Due to the symmetry of the system under rotation around the $z$ axis, one finds that the $z$ components of the Poynting vector do not vary in time. Hence, there is an asymmetry of the radiation emitted in the positive and the negative directions of the $z$ axis. This result proves that in spite of parity conservation by each element of the system, namely by the charge alone and by the monopole alone, the interference term of the radiation fields breaks parity.

It is not difficult to find the origin of the phenomenon described above. It was shown previously that magnetic fields of charges (15) and magnetic fields of monopoles (22) take opposite parities. The same is true for their electric fields. However, the energy-momentum tensor (21) takes the same form if the photon's fields of charges (15) are replaced by the photon's fields of monopoles (22). Hence, summing these fields in order to obtain the overall photon fields, one adds quantities having opposite parities and parity violation follows. Therefore, one should not be surprised to find parity violating properties whose geometric pattern resembles the one obtained by Wu *et al.* (1957) in the celebrated experiment that demonstrated parity violation in weak interactions.

The analysis carried out in this work proves, by means of an appropriate example, that parity nonconservation is an inherent property of a classical charge–monopole system. An essential element in the construction of a parity-violating system is the *correlated* motion of charges and monopoles. This motion yields interference that breaks the symmetry of electromagnetic waves emitted from the system at the two directions of the $z$ axis. It is interesting to note that the effect disappears if the charge and the monopole are fused into a single particle (dyon), because here no interference of radiation fields takes place.

## REFERENCES

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